<<偏微分方程数值解的有效条件数>>

图书基本信息

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章节摘录

Chapter 1E?ective Condition NumberIn this beginning chapter,new computational formulas are provided for the exective condition number Cond e?,and new error bounds involved in both Cond and Cond e? are derived. A theoretical analysis is provided to support some conclusions in Banoczi et al. [14]. For the linear algebraic equations solved by Gaussian elimination (GE) or the QR factorization (QR),the direction of the right-hand vector is insigni ¯ cant for the solution errors,but such a conclusion is invalid for the ¯ nite di- ?erence method for Poisson0s equation. The e?ective condition number is important to the numerical partial di?erential equations,because the discretization errors are dominant. The materials of this chapter are adapted from [95,127,134,138,218].1.1 IntroductionThe de ¯ nition of the traditional condition number was given in Wilkinson [227], and then used in many books and papers. To solve the overdetermined system of the linear algebraic equations Fx = b,the traditional condition number in the 2-norm is de ¯ ned by Cond = ?1=?n,where ?1 and ?n are the maximal and the minimal singular values of matrix F,respectively. The condition number is used to provide the bounds of the solution errors from the perturbation of both matrix F and vector b. The new computational formulas are derived in this chapter for the e?ective condition numbers,denoted by Cond e?,Cond E and Cond EE. The Cond e? denotes the enlarged factor of the solution errors over the residual errors. For further application,in this chapter we also derive the new error bound involved in both Cond and Cond e?. In this new bound,the Cond and the Cond e? denote basically the enlarged factors of the errors over the perturbation errors of matrix F and vector b,respectively. Hence,when the matrix and the vector errors are dominant,Cond and Cond e? will play an important role,respectively.First,we apply Cond e? to the linear algebraic equations,and a theoretical jus- ti- cation is given to support some conclusions in [14]. For the solution errors of the algebraic equations by Gaussian elimination (GE) or the QR factorization (QR),the direction of the right-hand vector is insigni ¯ cant. Then we apply Cond e? to the numerical partial di?erential equations (PDE) and the Tre?tz method (TM) for Poisson0s equation. Since the discretization errors are dominant,the e?ective condi- tion number is important. Moreover,since the TM solution is highly accurate,the small e?ective condition number well explains the high accuracy of the solutions, and strengthens the collocation Tre?tz method (CTM) in [155],where only error bounds and numerical experiments are provided without the stability analysis. It is explored in this chapter and the entire book that the huge Cond is often mislead-ing,but the e?ective condition number is the appropriate criterion for the stabilityanalysis.Here let us illustrate the references of the condition number and the e?ective con-dition number. The traditional condition number Cond was ¯ rst given in Wilkinson [227],and then used in many textbooks,such as Strang [197],Atkinson [3],Schwarz [191],Datta [44],Gill et al. [65],Golub and van Loan [66],Stewart [195],Horn and Johnson [90],Sun [199],Wang et al. [212],and Higham [88]. The condition num-ber for eigenvalues is introduced in Parlett [180] and Frayss?e and Toumazou [61], and there are more discussions in Gulliksson and Wedin [71],and Elsner et al. [55]. The Cond is applied to the numerical partial di?erential equations in Strang and Fix [198],Quarteroni and Valli [184] and Li [115]. On the other hand,the e?ective condition number was de ¯ ned and studied in Chan and Foulser [27]. However,its algorithm was ¯ rst proposed in Rice [186] in 1981,but the natural condition number was called. Only a few reports,such as Christiansen and Hansen [37],Christiansen and Saranen [38],Banoczi et al. [14],and Axelsson and Kaporin [7,8],follow the line of the e?ective condition number. The error estimates for the solution of the linearal gebraic system in Brezinski [20] are also related to the e?ective condition number.Recently,the new computational formulas of e?ective condition number have been studied and applied to symmetric and positive de ¯ nite matrices,which are obtained from Poisson0s equation by the ¯ nite di?erence method (FDM) in [127].